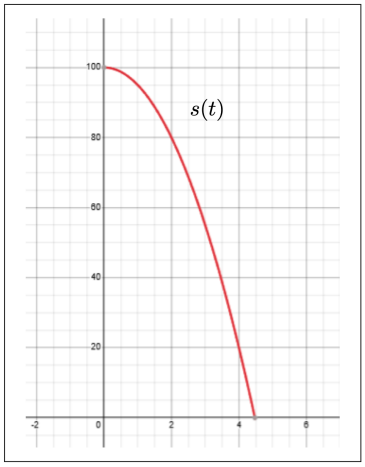
Sample Group Worksheet: Rates of Change

Calculus I recitation activity — Written by Dilan Karagüler, Lead TA, Fall 2025

This worksheet is meant to be done in groups of about four students. Talk the problems through as a group; everyone writes up their own answers.

The concept of rate of change is one of the most fundamental ideas in mathematics, appearing in many forms across history. Ancient mathematicians, from the Babylonians to Archimedes, were already exploring early notions of change: the speed of moving objects, the growth of populations, and the accumulation of quantities. Understanding rate of change lets us describe how one quantity responds to another, whether it is the speed of a falling rock, the temperature of a reaction, or the flow of water in a tank. Today this idea is the foundation of calculus, physics, economics, and beyond.

Problem 1 (A falling ball). A ball is dropped from the top of a building, and its height (in meters) above the ground after t seconds is $s(t) = 100 - 5t^2$.



(a) Find the average rate of change of the ball's height on $[0, 2]$. Interpret your answer, including units.

(b) Find the instantaneous rate of change of the height at $t = 2$. Comparing with (a), what does this tell you about the motion of the ball?

(c) With what velocity does the ball strike the ground?

Problem 2 (A chemical reaction). In a laboratory experiment, the temperature (in °C) of a mixture is modeled by $C(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 10$, where t is the time in minutes since the start of the experiment.

(a) Find $C'(t)$. What does it represent in this context?

(b) Compute the instantaneous rate of change at $t = 1$ and interpret it.

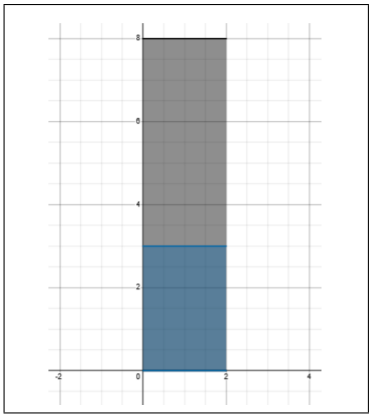
(c) When is the temperature not changing?

(d) For which t is the temperature increasing, and for which is it decreasing?

Problem 3 (An ancient water tank). Ancient engineers relied on water tanks and aqueducts to supply cities, irrigate farmland, and run public baths. They had to make sure the tanks neither overflowed nor ran dry too quickly. An engineer monitoring a tank over the first two weeks of summer finds that the water height is well modeled by

$$h(t) = 3 + 2\sin\left(\tfrac{\pi}{6}\,t\right), \qquad 0 \le t \le 14,$$

with h in meters and t in days. You can explore this function using the Desmos link.



https://tinyurl.com/132-water

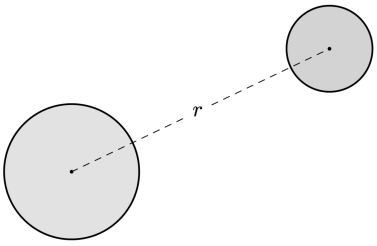
(a) What was the average change in the water height over the first week, $[0, 7]$?

(b) How is the water height changing at $t = 2$? Is the level rising or falling?

(c) Compute the acceleration at $t = 2$. Interpret it and relate it to (b).

(d) At what times is the water level momentarily not changing?

Problem 4 (Gravity). The gravitational force (in newtons) between two objects can be modeled as $F(r) = \dfrac{k}{r^2}$, where r (positive) is the distance between them in meters and the positive constant k depends on the masses.



(a) Find $F'(r)$. What does its sign tell you about how the force changes as the distance grows?

(b) With $k = 1000$, find the average rate of change of the force as r goes from 5 to 10.

(c) With $k = 1000$, find the instantaneous rate of change at $r = 5$.

(d) With $k = 400$, at what distance is the instantaneous rate of change equal to -2 N/m?